Adaptive Batch Size for Safe Policy Gradients

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Problem

- **Monotonically** improve a parametric **gaussian** policy $\pi_{\boldsymbol{\theta}}$ in a **continuous** MDP, avoiding unsafe **oscillations** in the expected performance $J(\boldsymbol{\theta})$.
- Episodic Policy Gradient:
  - estimate $\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$ from a **batch** of N sample trajectories.
  - $\boldsymbol{\theta}' \leftarrow \boldsymbol{\theta} + \Lambda \widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$
- Tune step size α and batch size N to limit oscillations. **Not trivial**:
  - Λ: **trade-off** with speed of convergence $\leftarrow$ adaptive methods.
  - N: **trade-off** with total learning time $\leftarrow$ typically tuned by hand.
- **Lack** of cost sensitive solutions.

Contributions

1. We propose a per-component adaptive step size Λ which results in a greedy **coordinate descent** algorithm, improving over existing *safe* adaptive step-size methods.
2. We show a **duality** in the role played by Λ and N in maximizing the performance improvement $J(\boldsymbol{\theta}') - J(\boldsymbol{\theta})$ and how a **joint optimization** of the two meta-parameters can guarantee *monotonic improvement* with high probability.
3. We make a first step in the development of **practical methods** to jointly optimize the step size and the batch size.
4. We offer a preliminary **empirical evaluation** of the proposed methods on a simple control problem.

Non-Scalar Adaptive Step Size

Lower bound to policy performance: [Pirotta et al., 2013]

$$J(\boldsymbol{\theta}') - J(\boldsymbol{\theta}) \geq \underbrace{\frac{1}{1-\gamma}\int_{\mathcal{S}} d_{\mu}^{\pi_{\boldsymbol{\theta}}}(s)\int_{\mathcal{A}}\left(\pi_{\boldsymbol{\theta}}(a|s) - \pi_{\boldsymbol{\theta}'}(a|s)\right)Q^{\pi_{\boldsymbol{\theta}}}(s,a)\mathrm{d}a\mathrm{d}s}_{\text{weighted advantage}}$$

$$-\underbrace{\frac{\gamma}{2(1-\gamma)^2}\left\|\pi_{\boldsymbol{\theta}'} - \pi_{\boldsymbol{\theta}}\right\|_{\infty}^2}_{\text{state distribution error}}\left\|Q^{\pi_{\boldsymbol{\theta}}}\right\|_{\infty} = B_L(\boldsymbol{\theta}', \boldsymbol{\theta})$$

Goal: $\boxed{\Delta\boldsymbol{\theta}^* \in \arg\max_{\Delta\boldsymbol{\theta}} B_L(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}, \boldsymbol{\theta})}$

- **Gaussian policy:** $\pi_{\boldsymbol{\theta}} \sim \mathcal{N}(\phi(s)^{\mathsf{T}}\boldsymbol{\theta}, \sigma^2)$
- **Gradient update:** $\Delta\boldsymbol{\theta} = \Lambda\nabla_{\boldsymbol{\theta}} J_{\mu}(\boldsymbol{\theta})$ with $\Lambda = \operatorname{diag}(\alpha_1, \alpha_2, \dots, \alpha_m) \geq 0$

If $\Lambda = \lambda I \rightarrow$ [Pirotta et al., 2013]

Solution: *coordinate ascent*

Exact Framework

Optimal step size:

$$\alpha_k^* = \begin{cases} \frac{1}{2c} & \text{if } k = \arg\max_i |\nabla_{\theta_i} J_{\mu}(\boldsymbol{\theta})|, \\ 0 & \text{otherwise} \end{cases}$$

$$c = \frac{RM_{\phi}^2}{(1-\gamma)^2\sigma^2}\left(\frac{|\mathcal{A}|}{\sqrt{2\pi}\sigma} + \frac{\gamma}{2(1-\gamma)}\right)$$

Improvement guarantee: $J(\boldsymbol{\theta}') - J(\boldsymbol{\theta}) \geq (4c)^{-1}\left\|\nabla_{\boldsymbol{\theta}} J_{\mu}(\boldsymbol{\theta})\right\|_{\infty}^2$

Approximate Framework

Given a policy **gradient estimate** $\widehat{\nabla}_{\boldsymbol{\theta}} J_{\mu}(\boldsymbol{\theta})$ s.t. $P\left(\left|\nabla_{\theta_i} J_{\mu}(\boldsymbol{\theta}) - \widehat{\nabla}_{\boldsymbol{\theta}_i} J_{\mu}(\boldsymbol{\theta})\right| \geq \epsilon_i(N)\right) \leq \delta$

Optimal step size:

$$\alpha_k^* = \begin{cases} \dfrac{\left(\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty} - \epsilon\right)^2}{2c\left(\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty} + \epsilon\right)^2} & \text{if } k = \arg\max_i |\widehat{\nabla}_{\theta_i} J(\boldsymbol{\theta})|, \\ 0 & \text{otherwise} \end{cases}$$

Improvement guarantee: $J(\boldsymbol{\theta}') - J(\boldsymbol{\theta}) \geq \left(\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty} - \epsilon\right)^4 (4c)^{-1}\left(\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty} + \epsilon\right)^{-2}$

Adaptive Batch Size

Idea:
- There are evidences that it is possible to adapt the batch size instead of the step length [Pirotta and Restelli, 2016, Bollapragada et al., 2017, Smith et al., 2017]
- In particular, in RL the cost of collecting new samples may be huge
- Larger step size $\rightarrow$ less parameter updates $\rightarrow$ lower costs

Cost-sensitive joint optimization

$$\boxed{\{\Lambda^*, N^*\} \in \arg\max_{\Lambda, N} \frac{B_{\delta}(\Lambda, N)}{N}}$$

e.g., bound on the estimation error with N samples (see approx. framework)

	Chebyshev	Hoeddfing	Empirical Bernstein [Mnih et al., 2008]
d_{δ}	$\sqrt{\frac{Var[\widehat{\nabla}_{\theta_i} J(\boldsymbol{\theta})]}{\delta}}$	$\boldsymbol{R}\sqrt{\frac{\log 2/\delta}{2}}$	$\sqrt{2S_N \ln 3/\delta}$
f_{δ}	$\times$	$\times$	$3\mathbf{R}\ln 3/\delta$

Chebyshev-like Bounds

Error bound: $\epsilon \leq \frac{d_{\delta}}{\sqrt{N}}$ with probability $(1-\delta)$

Optimal meta-parameters:

$$\alpha_k^* = \begin{cases} \frac{(13 - 3\sqrt{17})}{4c} & \text{if } k = \arg\max_i |\widehat{\nabla}_{\theta_i} J(\boldsymbol{\theta})| \\ 0 & \text{otherwise} \end{cases} \qquad N^* = \left\lceil \frac{(13 + 3\sqrt{17})d_{\delta}^2}{2\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty}^2} \right\rceil$$

Bernstein-like Bounds

Error bound: $\epsilon \leq \frac{d_{\delta}}{\sqrt{N}} + \frac{f_{\delta}}{N}$ with probability $1-\delta$

Optimal meta-parameters:

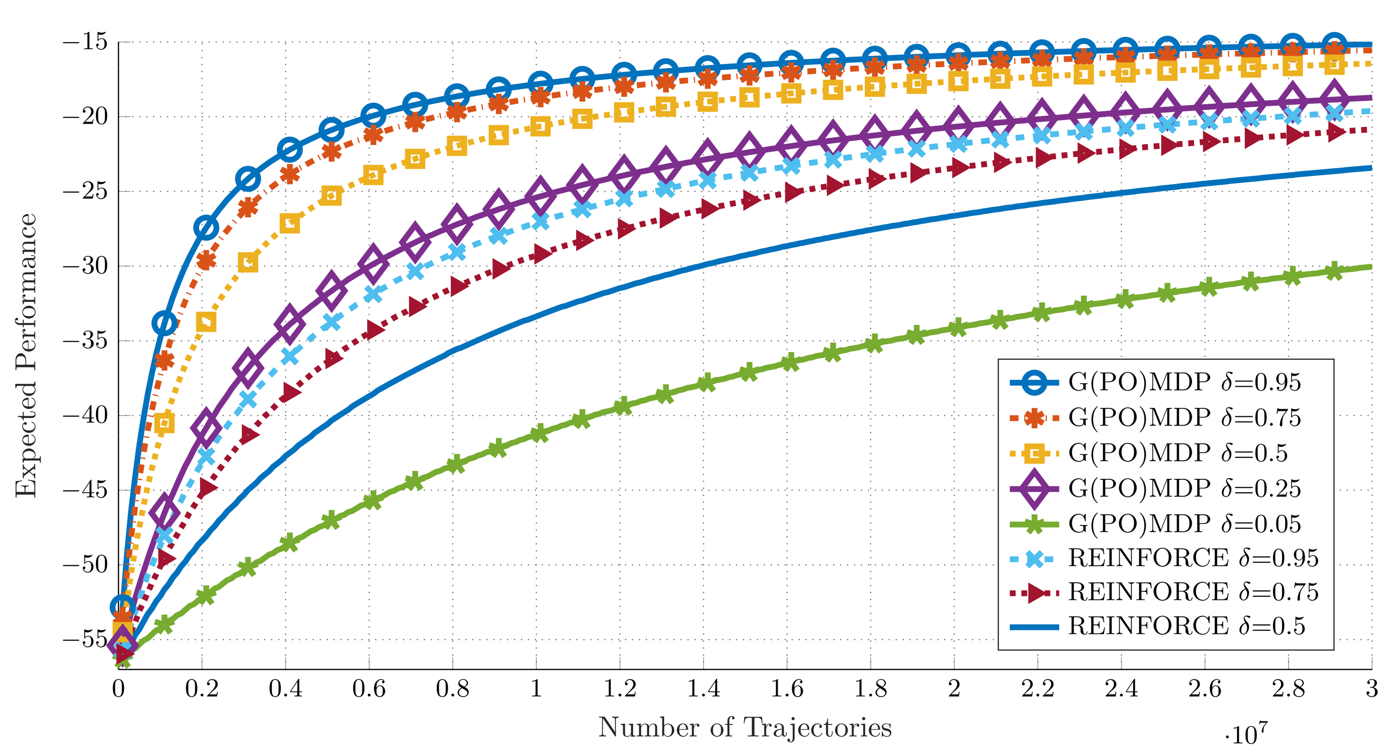
N^* has **no practical closed-form solution**: we **suggest** to find it with a linear search, knowing that:

- $N^* \geq N_0 \triangleq \left(d_{\delta} + \sqrt{d_{\delta}^2 + 4f_{\delta}\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty}}\right)^2 2^{-2}\left\|\widehat{\nabla}_{\boldsymbol{\theta}} J(\boldsymbol{\theta})\right\|_{\infty}^{-2}$
- the cost-sensitive objective is concave above N_0

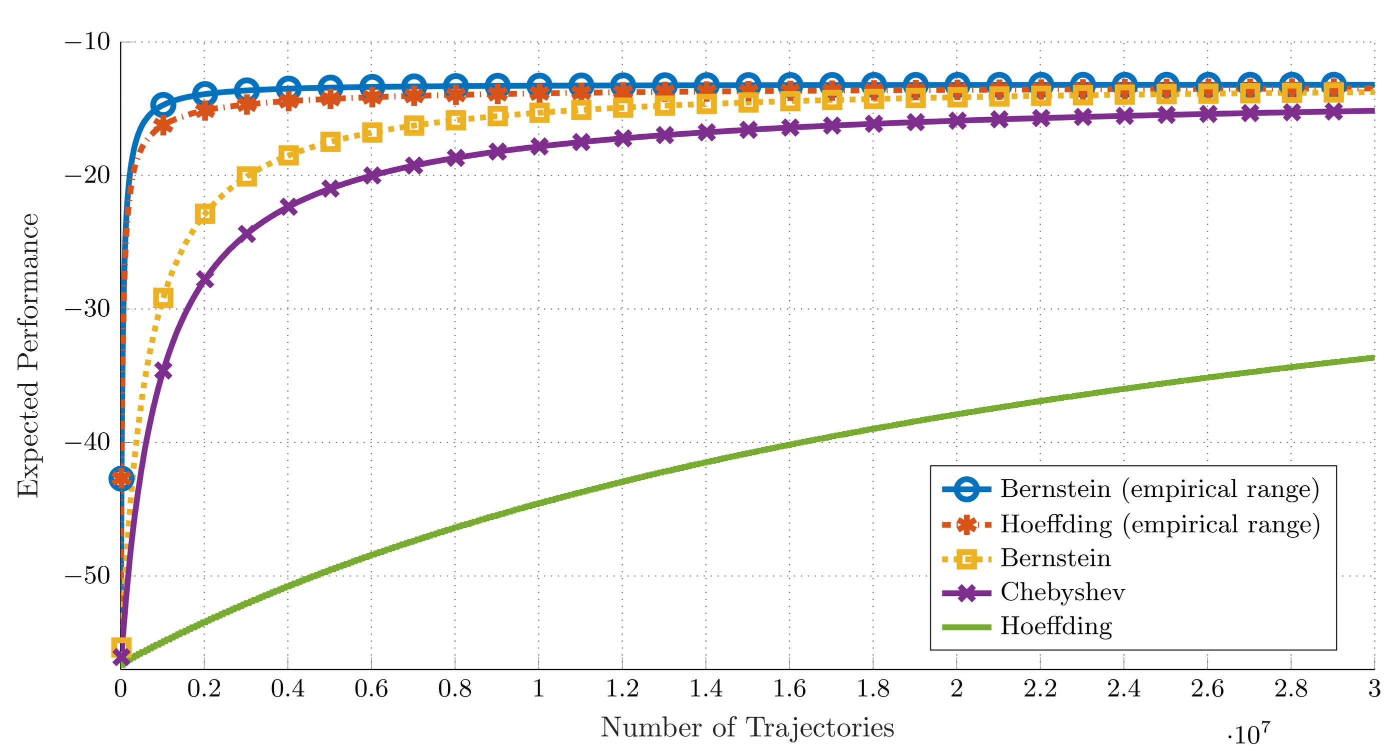
then compute Λ^* from $\epsilon(N^*)$

Empirical Results (One-Dimensional LQG)

Comparing gradient estimation algorithms and values of δ



Comparing statistical bounds (using G(PO)MDP and $\delta = 0.95$)



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